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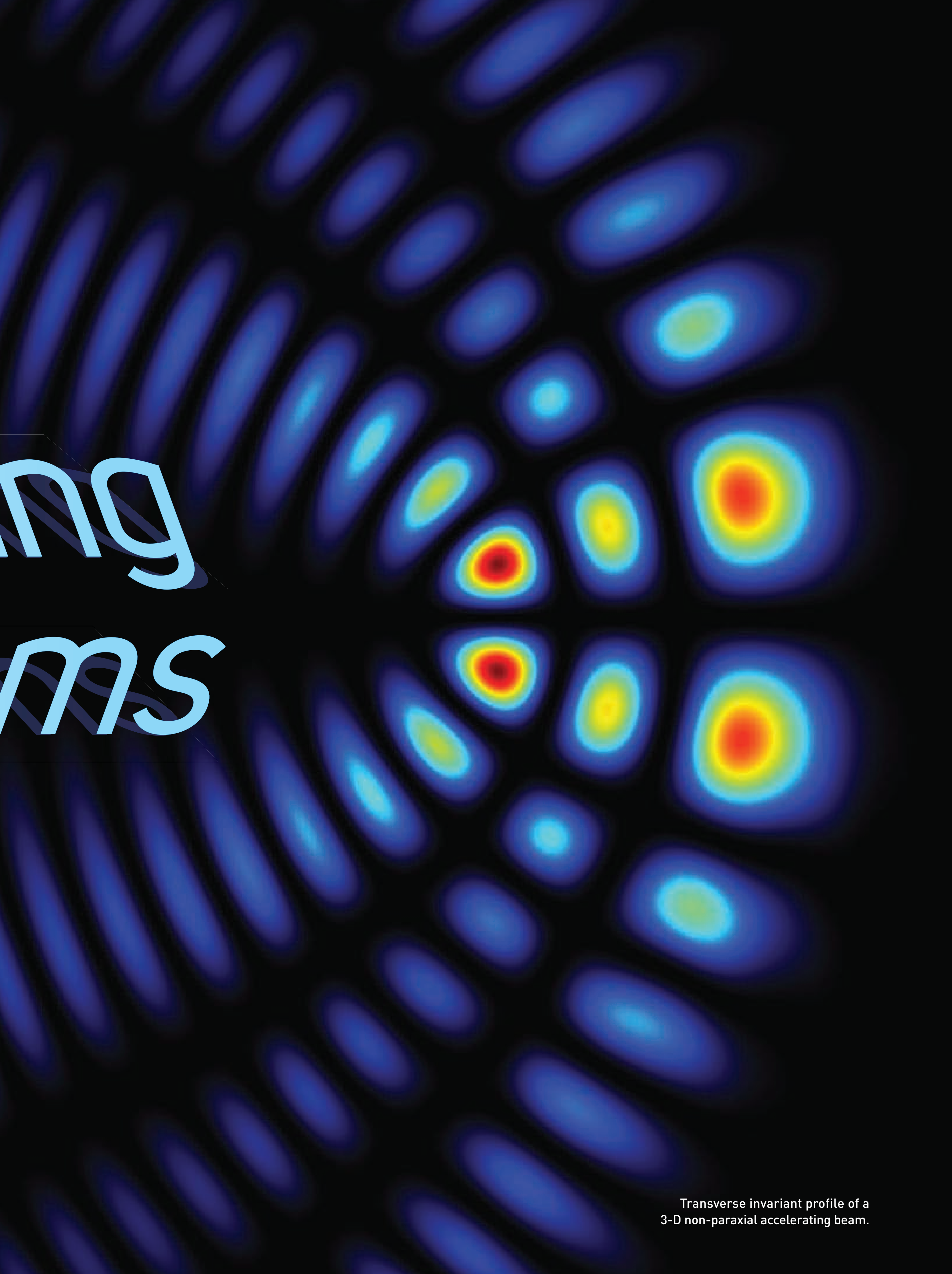
Accelerati
Optical Bea

Thanks to their unique interference, accelerating beams curve as they travel. They require no waveguiding structures or external potentials and appear even in free space. This beautiful phenomenon has led to many intriguing ideas and exciting new applications.



Transverse invariant profile of a
3-D non-paraxial accelerating beam.

The idea that light travels in straight lines has been ingrained in human consciousness since antiquity. Centuries after it was postulated by Euclid in his *Optica*, it became one of the pillars of Newton's corpuscular hypothesis. Even with the advent of wave theories and Maxwell's electrodynamics, this basic premise persisted as an outcome of electromagnetic momentum conservation.

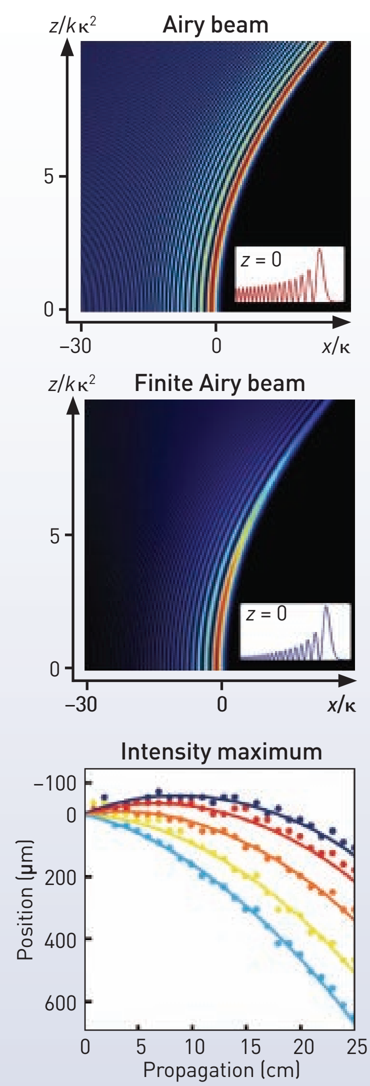
So it is only natural to expect that the intensity centroid of an optical beam or wavepacket will always follow a straight line. Yet nothing about this notion restricts how the specific intensity features of a beam will evolve during propagation, raising an intriguing question: Is it feasible to produce a shape-preserving optical beam whose intensity peaks move along curved trajectories? As exciting recent research shows, the answer is yes.

Remarkably, the intensity peaks of Airy beams follow parabolic trajectories much like the ballistics of projectiles.

Accelerating Airy beams

In 2007, researchers observed a new family of paraxial light beams—the optical Airy beams. Unlike ordinary optical wavefronts, Airy beams transversely accelerate (self-bend) throughout propagation. This exotic behavior, first encountered within the framework of quantum mechanics, is possible even in entirely homogeneous media such as free space. Remarkably, the intensity peaks of Airy beams follow parabolic trajectories much like the ballistics of projectiles.

For example, an Airy beam "launched" head-on will exhibit parabolic bending, but such a beam can also be launched at some upward angle (opposite to its direction of bending). In this case, the beam would decelerate and stall at its apogee until motion resumes downward due to acceleration. In this way, the main peak of Airy wave packets could circumvent opaque objects.



AIRY BEAMS

(Top) Ideal Airy beam (Center) Finite Airy beam; κ is an arbitrary transverse scale. (Bottom) Ballistic dynamics of the peak of finite Airy beams for various launch angles for the theoretical (solid lines) and experimental values (dots).

In an accelerated frame of reference ($x \rightarrow x + Az^2$), Airy beams are eigenmodes of the paraxial wave equation. As a result, they exhibit shape-invariant propagation in spite of their inherent bending. As a byproduct of their shape-preserving character, they exhibit remarkable resilience against perturbations and tend to regenerate during propagation after only a few diffraction lengths; this is called "self-healing," and it is useful for applications in turbulent and turbid systems or in hazy environments where scattering is an issue.

By its nature, the ideal shape-preserving self-bending Airy solution cannot be normalized. In other words, it carries infinite power, very much like a plane wave. Of course, in practice these non-spreading beams are truncated by an aperture and thus eventually diffract. If the geometrical size of the limiting aperture greatly exceeds the comparatively narrower spatial features of these propagation-invariant fields, the diffraction spreading process is considerably slowed over the intended propagation distance; hence, for all practical purposes, Airy beams retain their key characteristics: They maintain their intensity structure while their main intensity features ("lobes") bend along parabolic paths.

In all cases, their transverse centroid moves along straight lines—a clear manifestation of electromagnetic momentum conservation. This is possible because the power fraction contained in the long tail on the side opposite to the bending balances the momentum change that occurs due to the bending of the main peaks. Yet in many experimental situations, what

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plays a crucial role is the local light intensity (not the transverse power centroid), and thus this momentum balance is inconsequential. What actually matters in these cases is the fact that the intensity lobes of Airy beams do bend—which affects the outcome of experiments.

Interestingly, Airy beams represent the only possible shape-preserving, self-bending solution in paraxial systems with only one transverse dimension. However, in paraxial arrangements with two transverse dimensions, two separable families of solutions are possible: 1) two-dimensional Airy beams, constituted by one Airy function in each of the two transverse coordinates, and 2) accelerating parabolic beams, in which an innate parabolic symmetry is imposed. Moreover, any function on the real line can be mapped to an accelerating beam with a different transverse shape.

Given that paraxial propagation and linear chromatic dispersion obey similar evolution equations, one could also use Airy pulses as a means to overcome dispersion-broadening effects on temporal pulses. The very fact that Airy wavepackets represent the only non-dispersing one-dimensional solution in the paraxial regime allows them to be the sole building block that could be used toward synthesizing 3-D spatio-temporal bullets impervious to spatial and temporal spreading.

Nonparaxial accelerating beams

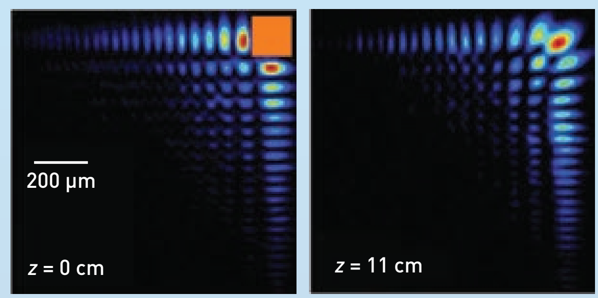
These shape-preserving accelerating beams were derived from the paraxial wave equation, which is a typical simplification of Maxwell's equations. It assumes that the angle between the optical axis and the wave vectors of the plane waves that constitute a beam is small enough that the wave does not deviate too much from its propagation direction. As a result, the transverse acceleration of the paraxial accelerating beams is restricted to small angles and forced to follow a parabolic trajectory.

This is a serious limitation, because spatial acceleration means that the propagation angle continuously increases, and eventually, after physically relevant distances, the beam trajectory reaches a steep angle beyond the paraxial regime. Moreover, paraxiality implies that the transverse structure of accelerating beams cannot have narrow lobes (small features), on the order of a few wavelengths or less. At the same time, reaching steep bending angles and having small-scale features is absolutely necessary in areas such as nanophotonics and plasmonics. Researchers have made several attempts to find non-paraxial accelerating beams, but the beams all

SELF-HEALING

Shape-preserving beams, such as accelerating beams and propagation-invariant beams (Bessel, Mathieu and parabolic beams), display what is called "self-healing:" If a section of the beam, particularly its main intensity maximum, is blocked by an obstacle, it reconstitutes itself after some propagation distance.

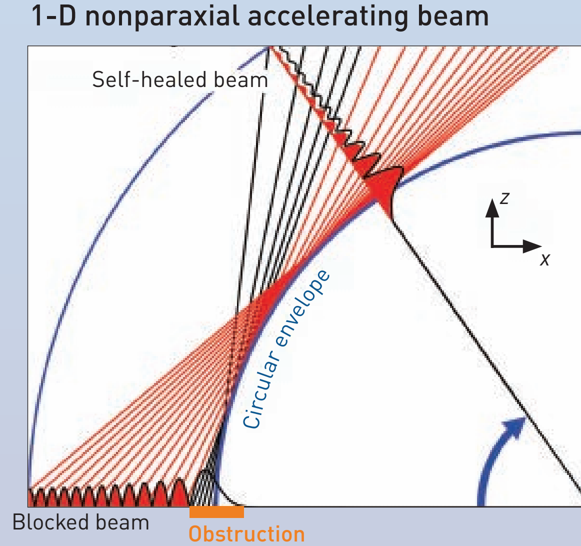
2-D Airy beam



Self-healing is easiest to understand in terms of a ray picture; the main spot of the accelerating beam is described as a bundle of rays forming an envelope near the intensity maximum.

At different propagation distances, distinct rays contribute to the maximum. Therefore, blocking a bundle of rays (black lines) only noticeably affects the intensity maximum within a limited propagation range; after that, the beam's intensity profile "heals itself" because all the required rays (red) are present. Self-healing reminds us that intensity maxima are not necessarily physical entities capable of transmitting information, and that this surprising behavior does not violate any fundamental physical law.

1-D nonparaxial accelerating beam



showed deformation and breakup; they were thought to be out of reach.

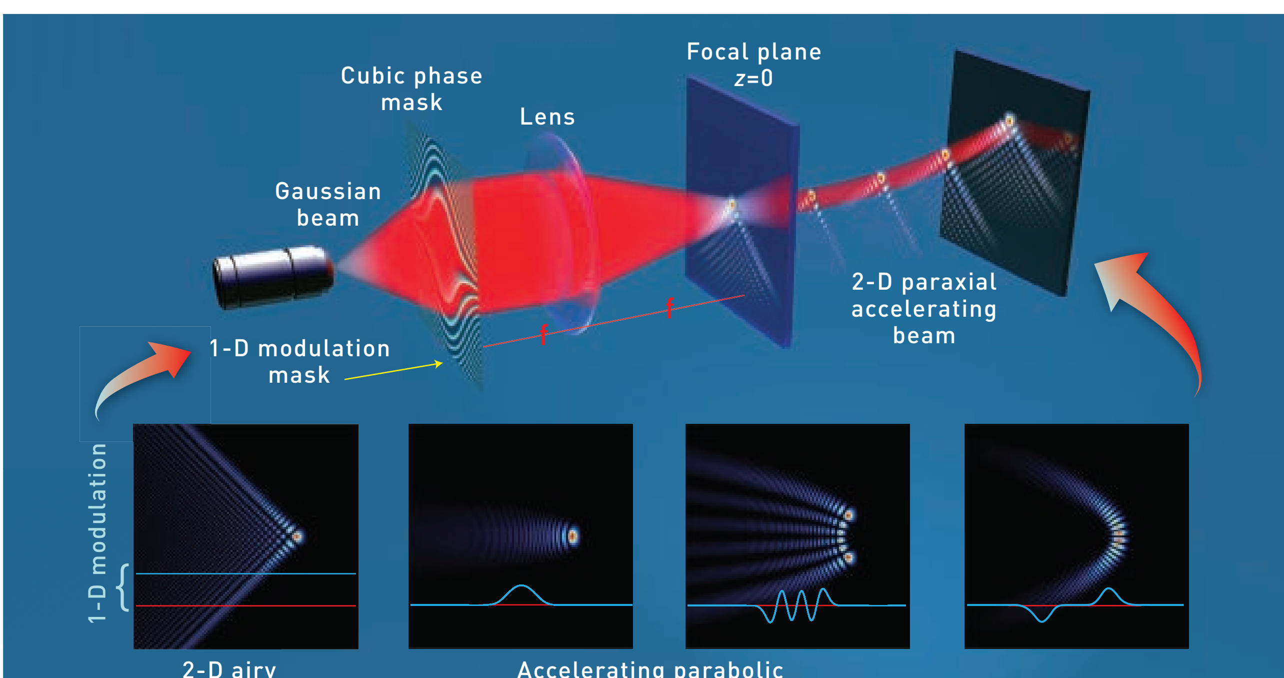
Recently, however, physicists have predicted and demonstrated non-paraxial accelerating beams that are exact solutions of Maxwell's equations. The theory of these beams is based on simple symmetry principles. Namely, the monochromatic scalar wave equation (Helmholtz equation) has no preferred axis. As such, searching for accelerating solutions that satisfy the circular symmetry of the equation guarantees invariance to rotations. Indeed, circular trajectories should be natural trajectories of accelerating beams of the full wave equation, scalar or fully vectorial.

The derivation was based on two logical steps: First, plane waves are superimposed along a closed circle in k-space to construct a wavepacket that is symmetric to rotation in the plane of propagation. This wavepacket has a Bessel profile, with the order of the Bessel function proportional to the radius of the accelerating trajectory.

Accelerating beams are formed by plane waves with different phase velocities, but with the proper phase distance to interfere in a specific way.

However, such solutions cannot be launched from a single plane: They are composed of both forward- and backward-propagating waves. This means that any solution that is exactly symmetric to rotation requires some of its wave constituents to come backward from the other side of the physical system. Thus, for a beam launched from a single plane, the second step of the derivation is to cut off all backward-propagating wave constituents. The resulting beam has a Bessel-like shape in half a circle.

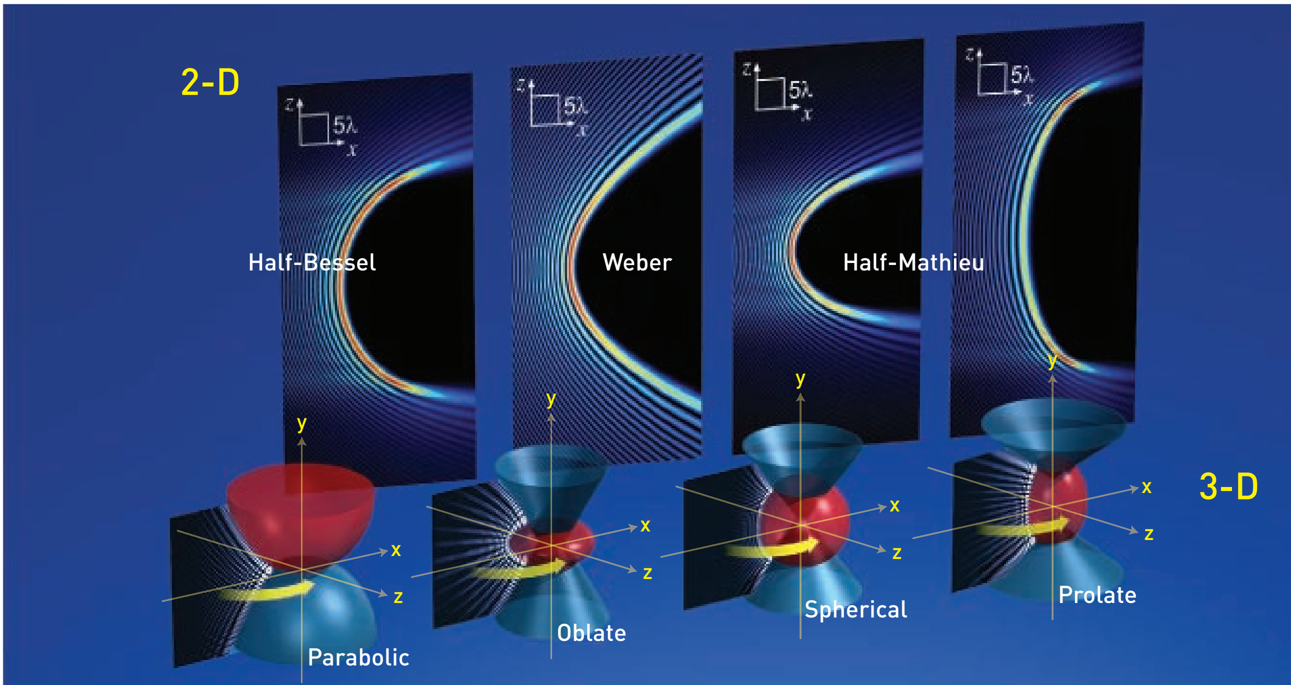
Consequently, the accelerating beam exhibits shape-preserving spatial acceleration along a circular trajectory, with the energy flux of each lobe displaying a turn of close to 90°. Moreover, given an initial tilt, such non-paraxial beams can bend to almost 180°. In fact, as we expect from basic wave physics, the polarization stays perpendicular to the energy flux—hence, such beams are both self-bending and self-rotating.



Paraxial generation of accelerating beams

In order to produce 1-D and 2-D accelerating beams, one can exploit the fact that their k-spectrum happens to be a cubic phase plus a 1-D amplitude modulation that encodes the transverse characteristics of the beam. For their synthesis, a plane wave or a Gaussian beam can be phase modulated using a cubic wavefront element and an amplitude mask with the 1-D modulation. The accelerating beam can then be easily generated at the focal point of a converging lens.

Phil Saunders/spacechannel.org



Non-paraxial accelerating beams

2-D (upper row) and 3-D (lower row) nonparaxial families of accelerating beams, propagating in the *z* direction. The blue and red surfaces represent the caustics of such beams.

Phil Saunders/spacechannel.org

Similarly, by utilizing other underlying symmetries of the Helmholtz equation, physicists have demonstrated new families of non-paraxial accelerating beams that travel along parabolic (Weber accelerating beams) and elliptic curves (half-Mathieu beams). Interestingly, since parabolas are not closed curves, the Weber accelerating beams naturally give forward propagating waves without any cutoff. Thanks to this unbroken symmetry, the local value of the linear momentum density of the Weber waves flows in parabolic lines, allowing the transfer of parabolic momentum to mechanical systems.

Remarkably, these new solutions to Maxwell's equations prove that the concept of wave acceleration appears in general electromagnetism. The property of acceleration is not a side effect of the paraxial approximation but something that is encountered in the full Helmholtz regime. Another important outcome is that exact shape-preserving beams no longer have to propagate only in a straight line; they can also be self-bending.

Here, it is important to stress the difference between non-diffracting beams and non-paraxial accelerating beams. In the circular case, both have a Bessel-like shape because, before the backward propagating wave constituents are cut, the solution is described by an exact Bessel function. However, this is just a mathematical similarity; the physical interpretation is different, as is the optical setup required to produce them.

Namely, Bessel beams are constructed through plane waves with the same phase velocity in the propagation direction. Such conical superpositions underscore their diffractionless property. On the other hand, non-paraxial accelerating beams are formed by plane waves with different phase velocities—but with the proper phase difference to interfere in a specific way.

Finally, researchers extended these ideas to three dimensions by studying solutions with 3-D rotational symmetry. These 3-D non-paraxial accelerating waves constitute a two-dimensional structure that exhibits shape-invariant propagation along semicircular trajectories. They can be classified and characterized in terms of spherical, parabolic and oblate/prolate spheroidal fields, from which these accelerating waves inherit their transverse structures. They do not have non-diffractive (Bessel-like) counterparts: The physics behind non-paraxial accelerating waves stands on its own.

Nonlinear accelerating beams

Many applications of accelerating beams have to do with the nonlinear interaction of light with some material—which has motivated many researchers to study the dynamics of accelerating beams inside nonlinear media.

In early work, the nonlinearity caused deformations in the accelerating beams and often led them to break up and emit solitons. Therefore, it was generally believed that accelerating beams cannot propagate in a shape-preserving manner inside nonlinear media.

This was proved wrong in 2011, when self-trapped accelerating beams were found to exist in Kerr and saturable media, as well as in the quadratic nonlinearity and in nonlocal nonlinearties. Soon after, scientists were experimentally observing nonlinear accelerating beams in Kerr media, quadratic media and in the deep non-paraxial regime.

Applications

Bringing the concept of Airy wave packets into the domain of optics has helped the scientific community to better grasp the physical phenomena of accelerating waves. More important, it has opened the door to new applications.

Particle and cell micromanipulation

Dholakia's group has exploited the optical forces exerted by accelerating waves to realize micrometer-sized "snowblowers" that could attract microparticles or cells at the bottom of a chamber and then blow them in an arc to another chamber. Thanks to the self-bending properties of the accelerating beams, neither microfluidic flow nor complex beam motion is required.

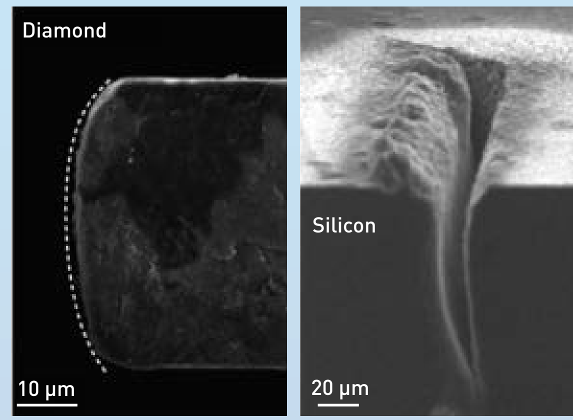
Laser micromachining

A key technological requirement in this area is that structures with longitudinally varying characteristics must be machined both at the surface and within the sample. This is challenging because it requires simultaneous control of beam steering and sample rotation and translation at micron scales. Dudley's group presented an elegant solution by using femtosecond accelerating beams to machine both curved surface profiles and trenches on micron scales.

For diamond and silicon samples, they successfully machined 50-μm-thick samples that had initially square edges into curved circular profiles with a 70-μm radius. The intense main peak of the accelerating beam was the one contributing to ablation while the rest of the beam freely propagated in air. They also used accelerating beams placed directly on silicon samples in order to process curved trenches that were roughly 80 μm deep within the bulk.

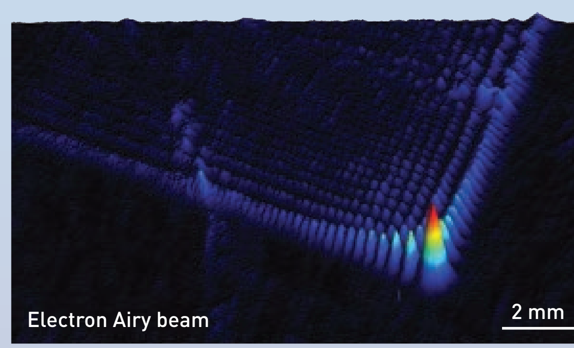
These results extended the technology of laser material processing into a regime in which curved features are generated from the intrinsic properties of the laser beam itself rather than from a geometric sample rotation.

APPLICATIONS OF ACCELERATING BEAMS



Micromachining using accelerating beams.

A. Mathis et al. Appl. Phys. Lett. **101**, 071110 (2012).



Experimental wave packet micrographs of 2-D electron Airy beams.

Courtesy of Noa Voloch-Bloch, Tel Aviv University

Light-induced curved plasma channels

When an ultra-intense laser pulse propagates in a dielectric medium such as air or water, it can ionize the atoms, thus creating plasma channels. This process, called filamentation, has myriad applications, including remote sensing, terahertz generation and lighting control. One of the important attributes of laser-induced filaments is the forward emission of broadband light, which can in turn be used for remote spectroscopic applications.

When filamentation is triggered by axially symmetric beams, the filaments are generated along straight lines. It is difficult to analyze the broadband radiation emanating from straight filaments, since emissions originating from different longitudinal sections overlap in the observation plane. To overcome this, Polynkin et al. showed that plasma channels generated through the filamentation of Airy beams were bent and followed curved trajectories. In this way, the broadband radiation emanating from different longitudinal sections of the curved filament followed angularly resolved paths, resulting in a spatial separation of the emission in the far field.

Self-bending electron beams

Recently, Arie's group experimentally generated accelerating beams of free electrons. To do so, they engineered the initial electron's probability density wave function to take on an Airy structure by using nanoscale holograms and magnetic lenses.

After that, the researchers observed that, upon propagation in free space, this Airy electron beam maintained the same shape and exhibited self-bending and self-healing like its optical counterpart. This type of beam could improve the resolution properties of tunneling electron microscopes.

Accelerating plasmons

Plasmon polaritons are electromagnetic waves that cling to the border between a dielectric material and a metal. In 2010, researchers introduced the idea of Airy plasmons; experimental observations soon followed.

From an applications perspective, Airy plasmons are important for the following reasons: First, diffraction poses a limit for the propagation of light on such surfaces, and the Airy plasmons are the only shape-preserving self-bending beams in two dimensions. Second, the self-healing property of the Airy wavepackets is especially useful on such surfaces, which are both absorbing and often suffer from defects. Third, the Airy plasmons are self-bending, giving us the means to control the flow of light on the surface.

Looking forward

It has been six years since the concept of accelerating beams was introduced to optics. The scope and variety of work that has emerged in that time has exceeded even the most optimistic expectations. Moreover, the concept of accelerating wavepackets has gone beyond optics and penetrated other fields, including plasmonics and electron beams.

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The reasons are three-fold. First, the concept is universal; not only does this phenomenon occur under paraxial conditions, it can be found for Helmholtz-type equations and higher dimensions, opening opportunities in other physical systems. Second, shape-preserving accelerating waves also occur in nonlinear systems, where the wave interacts with the medium and other waves. Third, early experiments have demonstrated how easy it is to experiment with these beams.

In view of this, it is natural to ask: what's next? It is already clear that self-bending wavepackets of matter waves are possible. Another intriguing area is ultrasonics; several groups are pursuing this direction. It is also interesting to explore whether the concept could apply in systems where the medium is not homogeneous.

Indeed, recently, accelerating beams have been found also in photonic lattices and crystals. Is it possible to generate accelerating beams in other environments? How about in random systems and heavily scattering media? Certainly, there are many more ideas to explore. Like the beams, the pace of research is only accelerating. OPN

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References and Resources

- G.A. Siviloglou and D.N. Christodoulides, Opt. Lett. **32**, 979-81 (2007).
- G.A. Siviloglou et al. Phys. Rev. Lett. **99**, 213901 (2007).
- M.A. Bandres. Opt. Lett. **34**, 3791 (2009).
- Kaminer et al. Phys. Rev. Lett. **106**, 213903 (2011).
- P. Aleahmad et al. Phys. Rev. Lett. **109**, 203902 (2012).
- M.A. Alonso and M.A. Bandres. Opt. Lett. **37**, 5175–7 (2012).
- F. Courvoisier et al. Opt. Lett. **37**, 1736–8 (2012).
- I. Kaminer et al. Phys. Rev. Lett. **108**, 163901 (2012).
- M.A. Bandres and B.M. Rodríguez-Lara, New J. Phys. **15**, 013054 (2013).